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Viewing Corona Pandemic with Mathematical Glasses: An Overview

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Abstract

The massive spread of the novel Coronavirus pathogen: COVID-19 is certainly one of the gravest situations mankind has faced in centuries. The fact that the virus spreads both, through direct (being in close contact with the patient) and indirect (through droplets that survive on the surfaces that have come in contact with the patient) exposure to an infected person, with an on-going research to confirm if the disease is air-borne as well, makes it all- the- more dangerous to humans. In this article, we aim at identifying some basic factors on which the rate of transmission of the virus depends, and the corresponding measures we can take in order to improve the situation.

Keywords

Corona virus, Exponential growth model, Logistic growth model, Physical distancing, Herd immunity

1. Introduction

With over 12.5 million people who have already been infected worldwide and a recorded 560 thousand deaths, the rate of spread of this disease is appalling. The situation in our own country is no different. Having nearly 800 thousand total cases and registering 22 thousand deaths, hitherto makes India the third worst affected country across the globe, after the USA and Brazil with 3.22 million+ and 1.76 million + cases, respectively.

This disease is supposed to have appeared for the very first time in Wuhan, province of Hubei, China, when a 55-year old man was found infected on November 17, 2019. Lack of knowledge about the symptoms, severity and necessary precautions led to a very rapid spread of COVID-19 in China, and eventually in the rest of the world. The known symptoms resemble that of seasonal flu like: dry cough, cold, fever and even shortness of breath in cases of acute infection. The symptoms get more troublesome as the virus affects the vital organs of the human body. As the general symptoms shown by the person infected with novel coronavirus overlap with common flu, it is difficult to identify the patients and hence the spread cannot be easily controlled *(Huang C. et al, 2020)*.

Dr. Robert Redfield, Director, Center of Disease Control and Prevention (CDC), in an interview with National Public Radio said that as high as 25% (or even a much higher percentage) of the number of people patients of COVID-19 may remain asymptomatic (*Cascella M. et al, 2020).* That is, they might just be walking around, unaware of the fact that they are actually infected and are spreading the virus. In the US, the fraction of asymptomatic people very quickly rose to 44% of the total cases, making this disease way more dangerous than the SARS-2002 (Severe Acute Respiratory Syndrome) outbreak *(Ksiazek, T. G et al, 2003).*

Researchers, all across the globe, are working round the clock to know more and more about the new pathogen in the interest of humankind. Administrations are also adopting all possible measures to control the pandemic and create awareness in general public in an effort to contain the transmission at the grass root level.

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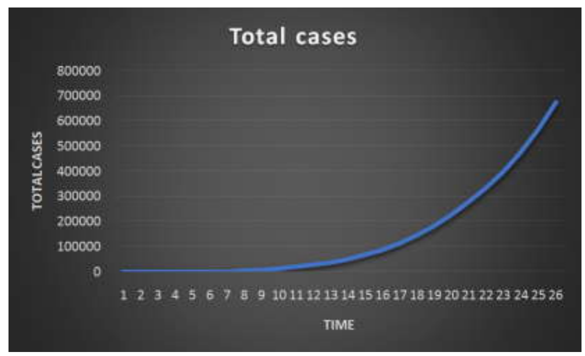
In the present article, we try to analyze the situation with a mathematical bent an. Scrutinizing the spread of COVID-19 from a mathematical point of view, we get to see that mathematical modeling and logistic and exponential growth models help us get to fairly conclusive results about the rate of reproduction, mortality and recovery. This article captures the model in Indian setting and context. The data and figures used would be in the sense of India, which would, in due course, also be valid for the global scenario.

2. Indian Scenario at present

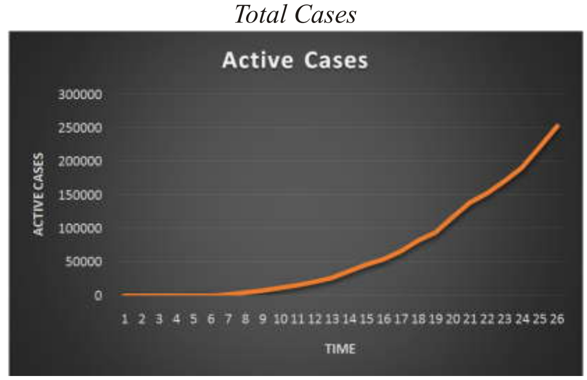
The first case of COVID-19 in India, which originated from China, was reported on January 30th, 2020. As of July 9th, we have a total of 767296 cases including 476377 recoveries and 21129 deaths, confirmed by the Ministry of Health and Family Welfare (MoHFW). The mortality rate due to Coronavirus stands at 2.80%, which is approximately 1.7 times lower than the world average, which is 4.70%. After the first case on 30th January, the 100th case was reported on 15th March, the 500th on 24th March, the 1,000th on 29th March, the 2,000th on 2nd April, the 5,000th on 8th April, the 10,000th on 14th April, the 50,000th on 7th May, the 100,000th on 19th May, the 200,000th on 3rd June, the 500,000th on 27th June and the 700,000th on 7th July, with Mumbai, Delhi, Ahmedabad, Chennai, Pune and Kolkata being the worst affected Indian cities.

Here are some figures concerning the total number of cases, active cases and deaths in India, which will help us plot and judge the behaviour of their respective graphs *(World Health Organization Report on COVID-19,* Aarogya Setu *Statistics, https://www.covid19india.org/, https://www.icmr.nic.in/) , https://mohfw.gov.in/ , https://www.who.int/).*

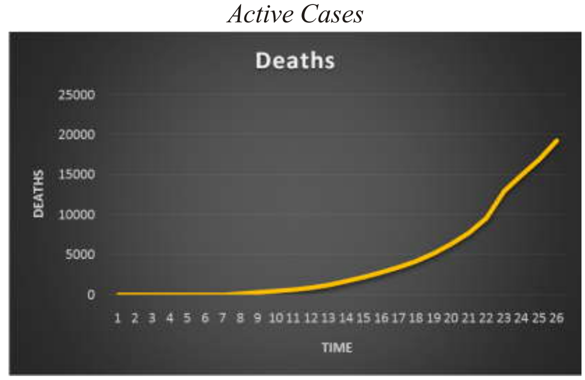
Day	**Total Cases**	**Active Cases**	**Deaths**
Day 1, 2nd March	5	3	0
Day 6, 7th March	34	31	0
Day 11, 12th March	74	69	1
Day 16, 17th March	137	126	3
Day 21, 22th March	360	365	7
Day 26, 27th March	834	794	19
Day 31, 1st April	1834	1792	41
Day 36, 6th April	4281	4267	111
Day 41, 11th April	7529	7189	242
Day 46, 16th April	12759	11214	420
Day 51, 21th April	18985	15460	603
Day 56, 26th April	26917	20486	826
Day 61, 1st May	35365	26027	1152
Day 66, 6th May	49391	35871	1694
Day 71, 11th May	67152	45925	2206
Day 76, 16th May	85940	53553	2752
Day 81, 21th May	112359	66089	3435
Day 86, 26th May	145380	82172	4167
Day 91, 31st May	182143	93349	5164
Day 96, 5th June	226770	116302	6348
Day 101, 10th June	276583	138069	7745
Day 106, 15th June	332424	152791	9520
Day 111, 20th June	395048	170269	12948
Day 116, 25th June	473105	190191	14894
Day 121, 30th June	566840	220546	16893
Day 126, 5th July	673165	253245	19268



Total Cases



Active Cases



Deaths

The results from these graphs show that the growth in the number of patients follows a fairly consistent pattern. Although initially we had an increase of only some tens of cases that eventually turned into about 20,000 cases per day, which seems to be a very huge difference or a severe deterioration in the condition, but checking the ratios of the corresponding days, we find an interesting result. This table represents the ratio of the cases on every fifth next day, which will help us in reaching certain results.

Day N v/s N+5	**Number of cases on day N**	**Number of cases on day N+5**	**Corres-ponding ratio**
Day 1 v/s Day 6	5	34	6.80
Day 6 v/s Day 11	34	74	2.18
Day 11 v/s Day 16	74	137	1.85
Day 16 v/s Day 21	137	360	2.63
Day 21 v/s Day 26	360	834	2.
Day 26 v/s Day 31	834	1834	2.20
Day 31 v/s Day 36	1834	4281	2.33
Day 36 v/s Day 41	4281	7529	1.76
Day 41 v/s Day 46	7529	12759	1.70
Day 46 v/s Day 51	12759	18985	1.49
Day 51 v/s Day 56	18985	26917	1.42
Day 56 v/s Day 61	26917	35365	1.31
Day 61 v/s Day 66	35365	49391	1.40
Day 66 v/s Day 71	49391	67152	1.36
Day 71 v/s Day 76	67152	85940	1.28
Day 76 v/s Day 81	85940	112359	1.31
Day 81 v/s Day 86	112359	145380	1.29
Day 86 v/s Day 91	145380	182143	1.25
Day 91 v/s Day 96	182143	226770	1.25
Day 96 v/s Day 101	226770	276583	1.22
Day 101 v/s Day 106	276583	332424	1.20
Day 106 v/s Day 111	332424	395048	1.19
Day 111 v/s Day 116	395048	473105	1.20
Day 116 v/s Day 121	473105	566840	1.20
Day 121 v/s Day 126	566840	673165	1.19

A careful examination of the last column of the table shows that the ratio ultimately attains a nearly constant value, which stabilizes and eventually lies between 1.2 and 1.4 after a certain period of time. This is a prime characteristic of the graph of the exponential growth model in entirety and also of the initial part of the graph of the logistic growth model, as detailed below *(Maier B. F et al ,2020 ; Stevens, H. ,2020).*

1. Exponential Growth Model v/s Logistic Growth Model for Coronavirus

For this discussion the term *birth rate* refers to the 'birth' of coronavirus, that it, from the time when it becomes active or attains the potential to infect humans, and correspondingly, *death rate* refers to the 'death' of coronavirus, that is, when it loses the ability to infect people (for example, when an infected person dies, and the dead body is disposed off completely, coronavirus present in the body can no longer infect any other person).

Exponential Growth Model: Consider a population (of COVID-19 pathogen) growth model. In exponential growth, we assume that the resources required for sustaining the population at any time are unlimited or infinite and will never run out. The growth can be approximated on the following lines:

Let N be the size of the population at the time of study, with α being the rate of birth, and β being the rate of death. Therefore over time T, the rate of change of population, denoted by $\frac{dN}{dT}$ is given by:

$$\frac{dN}{dT} = \alpha N - \beta N = (\alpha - \beta)N$$

or, $\frac{dN}{dT} = r * N$; where $r = (\alpha - \beta)$ and is commonly termed as *rate of reproduction.*

Depending on whether the value of r is positive or negative, we have a corresponding increase or decrease in the population.

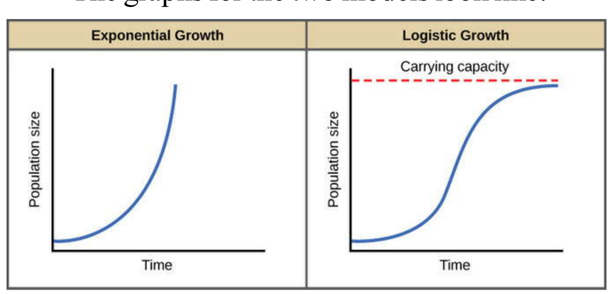
Logistic Growth Model: Exponential growth is a theoretical concept. What we usually find in real world scenarios is the logistic model. Consider a population (of COVID-19 pathogen in this case) growth model. In logistic growth, we assume that the resources required for sustaining the population at any time are limited and will run out eventually. The growth can be approximated on the following lines:

Let N be the size of the population at the time of study, with being the rate of birth, being the rate of death and k being the carrying capacity. Over time T, the rate of change of population, denoted by is given by:

$$\frac{dN}{dT} = (\alpha - \beta) * N * \frac{k - N}{k}$$

or, $$\frac{dN}{dT} = r * N * \frac{k - N}{k}$$

The graphs for the two models look like:



A careful visualization of the two graphs tells us that the exponential growth pattern has no non-zero equilibrium position for population, whereas the logistic growth pattern attains its equilibrium at a non-zero *k*, the carrying capacity of population. To know more on these growth patterns, one can refer Belinda Barnes and G. Fulford *(Belinda Barnes et al, 2002).*

Also, since infinite resources theory for Coronavirus in the real world is impractical, it is safe to believe that this pathogen also follows logistic growth trend, and the curve will ultimately plateau or level off after it reaches the inflation point.

The other thing we notice is that if the effective rate of growth of virus, taking all factors such as: recovery rate, mortality rate, etc., into account is denoted by *R*, we see that when $R>1$, the number of pathogens increases exponentially, consequently leading to an increase in the number of patients. The situation comes under control when *R* becomes unity, because that can be treated as a case when a person can spread the virus to only a single other person. In a similar fashion, when the value of *R* drops to <1, that is when the epidemic starts coming to an end.

Apart from studying the virus population, we can also study the human population by segregating them into three different categories: the susceptible group, the infected group and the removed group. 'Removed group' refers to the people who are either dead because of the disease, or are removed from the system due to the fact that they have recovered from the disease, and the antibodies so formed in them last a lifetime and they can never get infected again. An analysis of a similar kind is done by some researchers. *(Bhola J. et al, 2020).*

2. Capturing the factors involved

Suppose the total number of infected people on a particular day is . The number of people who are susceptible to the infection is directly proportional to , since more the number of patients in a particular closed environment, more are the chances of the healthy ones acquiring the infection.

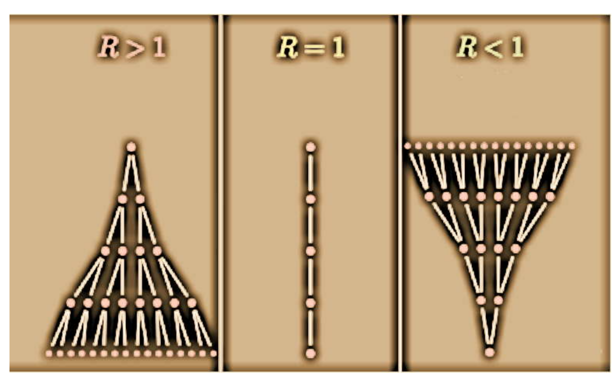
Further, every person who comes in contact with an infected person does not necessarily catch an infection. It is possible that a person with a very short exposure may catch the disease, whereas, possibly a person with a comparatively longer exposure might, as well, not catch it. Therefore, it would be absurd to assume that if a patient comes in contact with 100 people each day, then all 100 of them necessarily develop the disease. This fact leads us to consider an extra factor, called the *probability factor 'p'*, for having a fair approximation of the number of people who can get infected (*p* lies between 0 and 1). This clearly indicates that more the probability of a person catching an infection, higher would be the number of new people who become infected. Note that this probability would not be the same for all individuals. As the population is large enough, we ignore random differences among individuals and assume it to be constant *p* for every individual, i.e. every individual will have an equal chance *p* of getting infected while coming in contact with an infected person.

The last factor on which will probably depend is the amount of exposure an infected person has. That is, if a patient comes into direct or indirect contact with a larger number of people, then he has a chance of spreading the infection amongst a larger community. We consider *A* to be the average number of people who come in contact with a patient each day. This average *A* would also be different for different infected persons but for our analysis, we assume it to be a constant for all the infected persons. We, therefore, arrive at the final result that the number of new cases each day () would be directly proportional to the number of existing cases (), the probability of getting infected (*p*) and the average number of people who come in touch with the patient each day (*A*). Thus we have the following:

$$\Delta N_0 \propto \begin{cases} f(N_0) \\ g(p) \\ h(A) \end{cases}$$

where *f. g, h* are increasing functions of their respective variables.

[In essence, we have got that the number of new cases per unit time would be some function of , p and A that varies directly with either of these three. For instance, we could take as or or and so on for capturing the increase in infection, as all of these exhibit direct proportionality to the three variables. To decide the most appropriate proportionality, one needs to have requisite data in terms of the desired variables (which is difficult to be fetched at present) and a deeper analysis of the same. But for all these direct proportion-alities, the further discussion in this article stands valid].



To carry forward the discussion, let us take a particular direct proportionality and analyse the effect as under:

Assume that

$$\Delta N_0 = N_0 * p * A$$

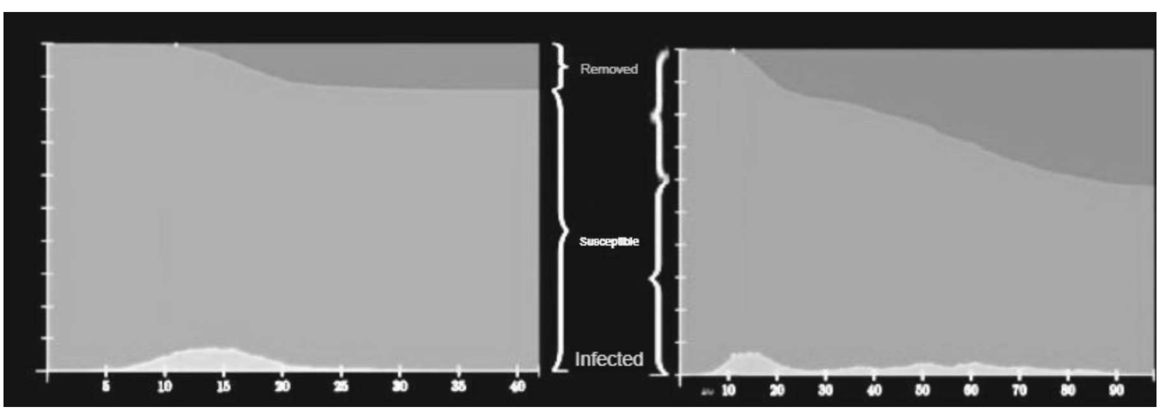
This seems quite alarming as it points that the cases will always be increasing and, in the worst case, the entire human population globally would get infected by the novel Coronavirus disease. However, there are possible way outs to avoid such a situation, and these way-outs lie in reducing the value of the constant as far as possible. Now, this can be achieved by minimizing either of p or A or possibly both *(https://3blue1brown.com)*.

Minimizing A means limiting the number of people who come in touch with the patient, and that is exactly why COVID-19 patients are kept under observation in isolation wards, with even the doctors, medical and para-medical staff members wearing PPE (Personal Protective Equipment) Kits before attending them. Further, lockdowns and curfews are also ways of reducing the value of A. So, we are sure about the utility of isolation as a part of the treatment, and lockdowns as effective means of preventing the spread, since the mathematical formula also points towards the same.

The second factor is minimizing p or the probability of getting infected. This can be done by maintaining proper hygiene, washing hands, reducing direct/ indirect physical contact with people as far as possible and avoiding crowded places. And we see, that is what exactly lockdowns aim at. Washing hands and sanitizing daily household items is another step towards minimizing p and eventually the rate of spread.

3. Physical Distancing and impacts

Talking about the physical distancing (what popularly came to be as *Social Distancing*) factor, as we already saw in the aforementioned formula that the number of new people developing the infection per day is very closely linked with the number of people whom the patient comes in contact with. And thereby, physical distancing plays a key role in deciding the fate of the susceptible population. But we also need to know that 10 out of 100 people following physical distancing and the remaining 90 of them moving ahead with their daily routines as usual, is as good as nobody doing anything. Or, for that matter, 90 people following the norms of distancing and just 10 people escaping it is also only a bit better than doing nothing at all since that will also keep the disease spreading, may be at a slower, but a consistent rate and we would never be able (or at least take considerably longer time) to get the disease eradicated completely. Here are two graphs which show how long does it take to get over the disease when a certain percentage of people start staying away from one another. The curves are plotted between the number of active cases on a particular day (Caution: the graph talks about active cases, and not about total cases) and the number of days it takes for the disease to vanish completely (*Simler, K. , 2020; https://www.3blue1brown.com ; https://www.numberphile.com/)*.



Physical Distance Factor
(100% of Population)
Disease Eradicated in 30 days

Physical Distance Factor
(90% of Population)
Disease Eradicated in 90 days

This is what happens when all the people follow physical distancing and avoid any kind of contact. The disease will get eradicated in close to 30 days, and less than 10% of the total population will be affected by it on any given day.

A little carelessness on the part of 10% of the population, and we see that the disease takes approximately 90 days to go away completely. We also notice that whereas in the first case, the curve representing the number of patients rose only once, till nearly 14 to 15 days after the first case was reported, and then followed a strictly downward trend, but when a few people do not take necessary precautions, then even after following the same trend initially, the graph again rises when we reach 35 days, 50 days, 60 days and 80 days. Apart from taking thrice the time to recover from the disease, the number of people who got infected by it also grew considerably.

We conclude that physical distancing is a powerful way to control the spread, but even a small number of people escaping the process will prolong the duration of the disease to an unimaginable extent.

The other thing we get from this is that if physical distancing is imposed at a very late stage, and a large number of people have already been affected, then it is virtually of no use. Taking a simple example, if a city where this disease spreads has 1,000 families, with an average of 6 members per family, and initially they do not follow any distancing, then say, at least one member of 100 families has already caught the infection. Even if we then start with very strict norms of physical distancing or home quarantine, even then, at least each family member of these 100 infected people is at a very high risk of developing the symptoms of the disease in a very short span of time. Additionally, however hard we try, there might be emergency situations wherein it becomes very essential for a person to travel. The person might spread infection in this process also when he steps out for something important and comes in contact with a certain number of people.

Apart from distancing and all other necessary precautions, there is yet another factor which plays a key role in the entire process. One very essential aspect on which probability of getting infected depends is the number of susceptible people in the given closed setting. When a large number of people have already got infected, their coming in touch with one another does not give rise to new cases. This is what brings us to the concept of '*herd immunity.*'

4. Herd immunity

When most (or a significantly large fraction) of a population has already been infected or is immune to an infectious disease (in case of diseases wherein the antibodies developed last a lifetime, and there is a negligible chance of the person catching the same disease again), there is an indirect protection of the body to the disease. In simpler words, a body that is already infected at present, does not have a possibility of catching an infection again. Therefore the number of susceptible people becomes very less, owing to the fact that most people have already been affected. This is what we call herd immunity (or herd protection) to those who are not immune to the disease (*D' Souza et al, 2020).* Due to herd immunity, the number of susceptible bodies goes down by a considerable amount, and resultantly, the number of new cases arising per day also gets lowered significantly.

As we know, the number of cases in the world are more than 12.5 million, that simply implies that these 12.5 million people (considering no chances of re- infection) or at least a large number of them (considering chances of re- infection or reappearance of symptoms of the same disease in the same person) are no longer susceptible to COVID-19, whether we consider the active cases (a person currently infected cannot get infected again), the recovered ones (considering the antibodies to provide a long term immunity to the person) or the fatalities.

Momentarily, it may strike to us that herd immunity can be the only possible situation for us to wait for in case of diseases like COVID-19, wherein we neither have vaccines for treatment, nor any kind of prior experience of dealing with this. But herd immunity is not achieved until a very large population has already been affected, and this large population may even be as large as 80 to 90% of the total, which in itself would be very alarming and dangerous. By the time we are waiting and relying on herd immunity to play its part and have the situation under control, the peak of the graph is already very high and the existing healthcare resources are already overwhelmed and supremely insufficient for a developing country like ours, excessive population being another constraint, when it comes to such a fatal and life threatening disease. The mortality rate would already have been very high by then. And that is exactly why we cannot wait for such a situation to occur.

5. Conclusion

The data that has been collected over the past few months probably hints that the number of cases will continue to increase, at least for a few more weeks, before we can hope to see a decline in the number of cases. But, as we come to the end of the article, we clearly understand that physical distancing, avoiding unnecessary travels and proper hygiene are the only possible way outs to protect ourselves from this deadly disease at present. Co-operating with doctors, medical and para- medical staff and following the safety guidelines issued by the government in the interest of public are our prime responsibilities. Mathematicians, chemists and biologists are all striving extremely hard to come up with ways that can save the world. Till the time the situation is not improving, we need to have faith in science, and take all necessary precautions.

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